International Journal of Theoretical Physics, Vol. 27, No. 1, 1988

Towards a Combinatorial Description of Space and Strong Interactions

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Received July 27, 1987

A reinterpretation is given of a successful phenomenological approach to hadron self-energy effects known as the unitarized quark model. General arguments are given that the proper description of strong interactions may require abandoning the assignment of a primary role to continuous concepts such as position and momentum in favor of discrete ones such as spin or W-spin. The reinterpretation exploits an analogy between the W-spin diagrams occurring in the calculations of hadronic loop effects and the spin network idea of Penrose. A connection between the S-matrix approach to hadron masses and the purely algebraic approach characteristic of the quark model is indicated. Several hadron mass relations generated by a resulting $SU(6)_W$-group-theoretic expression are presented and discussed. Results of an attempt to generalize the scheme to the description of hadron vertices are reported.

1. INTRODUCTION

Despite important successes of local field theory, the fundamental questions concerning the precise connection between the classical macroscopic space-time satisfying Einsteinian locality and the nonlocal properties of quantum theory are still unresolved. Thus, although quantum chromodynamics offers the possibility of describing the world of hadrons in terms of a local quantum field theory of quarks and gluons, one may still argue that in a more fundamental quantum approach to strong interactions one should not assume the classical space-time continuum as one of the primary concepts of the theory.

In accordance with the latter idea, the present paper proposes a different interpretation of the successful phenomenological approach to the question of hadronic self-energy effects studied recently (Żenczykowski, 1986;

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Törnqvist, 1979). The success of the mass formulas derived within this approach (Żenczykowski, 1986) is here attributed to a suggested tight connection between strong interactions and the supposed quantum origin of macroscopic space. In addition, this paper contains more complete meson mass formulas, discussions on the generality of mass relations obtained previously, and results of an attempt to apply the scheme to the description of hadron vertices.

General arguments supporting the view that the problem of strong interactions is perhaps closely related to the supposed quantum origin of classical macroscopic space are presented in Section 2. In this section the spin network idea of Penrose is briefly recalled. The point of view is adopted that it constitutes a part of a more complicated discrete network, from which macroscopic space and strong interactions properties should be derived.

In Section 3 a simple algorithm allowing the calculation of the combined spin-flavor dependence of meson and baryon mass differences is given. This algorithm, suggested by S-matrix considerations, is interpreted in the spirit of the discrete quantum network idea.

An attempt to apply the scheme to the description of vertex symmetry breaking is briefly reported in Section 4.

Finally, the standard interpretation of the phenomenological approach of Żenczykowski (1986) and Törnqvist (1979) and the reinterpretation of this paper are juxtaposed in the last section.

2. GENERAL

In quantum field-theoretic approaches space-time provides a classical background, intuitively thought of as a medium in which the particles propagate according to the rules of quantum theory. A prevalent opinion is that the concept of classical space starts to be in conflict with quantum theory below $\sim 10^{-33}$ cm only, where quantum gravitation effects should be considered. This is much below $\sim 10^{-13}$ cm, the characteristic distance of strong interactions, and $\sim 10^{-16}$ cm, the distance at which QED has been experimentally verified. It may seem therefore that the idea of linking strong interactions with the quantum origin of space is not tenable. It is known, however, that quantum theory possesses nonlocal properties (Bell, 1964; Stapp, 1971; Clauser and Shimony, 1978). Furthermore, the experiment of Aspect *et al.* (1982) has confirmed that physical systems may exhibit strongly nonlocal features (over macroscopic distances of order 10 m). The experimental results are both in excellent agreement with quantum theory and in violent disagreement with conventional (as drawn from special relativity) ideas about the propagation of causal influences. Consequently, the opinion is more and more often expressed (Clauser and Shimony, 1978) that our view on the nature of space-time requires a thorough revision. Since the problem occurs already for flat space-time, it seems that this conflict between quantum theory and our concept of space should be resolved before one attempts to take gravitation into account. Thus, the Planck length need not correspond to any critical distance at which one might expect the elucidation of the origin of the conflict between quantum rules and our understanding of space-time.

Similarly, the distance 10^{-16} cm obtained from the experimentally verified application of local field theory does not set an upper limit for such a critical distance. In fact, both the algorithm of quantum theory and the Aspect experiment suggest that no such critical distance exists.

In the framework of local field theories both classical (e.g., continuous space-time) and quantum (e.g., quantization prescription, rules of calculation) concepts are built in as primary concepts. Within such theories it is therefore impossible to study the conceivable possibility that they are actually related. It has been proposed in various contexts (Patton and Wheeler, 1975; Wheeler, 1973; Chew, 1971; Penrose, 1971) that continuous classical space-time should be considered secondary and that the primary concepts should be quantum-theoretic and most likely discrete (Penrose, 1971). The points of space-time would then be derived concepts. The most obvious physical concept which has well-known discrete (as opposed to continuous) quantum properties and is tightly connected with the notion of space-time itself is angular momentum. The idea of Penrose (1971) was therefore to start from angular momentum and build from it the concept of space in some way. His basic idea begins with the problem of defining the direction of spin projection of, say, a spin-$1/2\ \hbar$ particle. Such a particle has only two "directions" to choose from [forget about the direction provided by the continuous (!) momentum]. Penrose writes: "whether these possibilities are 'up' and 'down' or 'right' and 'left' depends on how things are connected with the macroscopic world. Since we do not want to think of such alternatives as referring to preexisting directions of a background space—we must deal with total angular momentum j only." The relative orientation of such a spin-j particle may be defined only with respect to some larger (higher j value) structure (which thus could be thought of as quasiclassical) belonging to the discrete system under consideration. This orientation can be determined through an "experiment" in which the particle and the larger structure examining it combine and/or exchange spin. This is a typical situation encountered in quantum systems: the outcome of an experiment depends both on the examined object and on the whole experimental setup. In this way one is led to the study of spin networks and the closely related $3n-j$ symbols, which should be among the basic structures of the combinatorial approach to macroscopic space.

In the simplest version of his approach Penrose considered a universe built out of such spin networks and attempted to define angles and hence rotations in terms of these spin structures. Later he was led to the study of conformal group in which rotations and translations are treated on an equal footing (Penrose and MacCallum, 1973). Although the resulting scheme was originally intended to approach in a different way the problem of the quantization of gravity, attempts to apply it to elementary particle physics and in particular to strong interactions have also been made (Perjés, 1979; Penrose *et al.*, 1978). The proposed particle classification (Penrose, 1977; Perjés, 1977) does not seem, however, to correspond in a direct way to the standard pattern in which elementary "particles" are grouped into generations built from leptons and three-colored quarks, both species coming in weak-isospin doublets. Such a correspondence, with quarks as conformal semispinors, can be obtained (Budini, 1979) only at the expense of enlarging the original conformal group, an indication that the way of combining rotations and translations in the original twistor scheme may not be wholly correct nor sufficient. Putting aside this scheme as a whole, it may still be argued that the spin network idea should be very well suited to the description of strong interactions in which spin seems to play a very important role (Anonymous, 1985; Grandpeix and Lurçat, 1985). Note also that no distance scale is fixed by the spin network alone. Thus, it should be possible to introduce the hadronic characteristic scale of 10^{-13} cm into the theory. In this paper our attention is restricted to spin structures only. No constructive proposal for any "discrete predecessor" of momentum (distance) is made. It is thought, however, that "individual particles and simple systems would not really know what momentum is" (Penrose, 1971). This should be contrasted with the *S*-matrix approach, which dispenses with the notion of space-time but retains the concept of continuous classical momentum (Chew, 1971).

The aim of this paper is to propose a combinatorial expression for the spin dependence of hadron masses. The association of the problem of mass with the problem of strong interactions may seem unjustified at present. However, the clarity of the concept of mass is somewhat blurred in strong interactions. In standard approaches one first assumes the existence of pointlike quarks within hadrons and assigns them current mass as if they were free ordinary particles like leptons. Then, through a confinement dogma these "particles" become unobservable and the "long-distance," "constituent" quark masses[2] are believed to emerge as more appropriate in the

[2]The *best* available *parameter-free* model for baryon magnetic moments (from which constituent quark masses were originally determined) is the model of Schwinger (1967; Żenczykowski, 1985), in which the "constituent" quark mass is *by definition* half the mass of the corresponding vector meson.

description of many low-energy properties of hadrons. Setting apart any specific implementation of this general scheme, it is obvious that any such scheme is composed of two logical steps, the second of which (confinement) is contrived to cancel partially the assumption (assignment of mass to a quark) made in the first step. If the concept of mass cannot be applied to a single quark (not even zero mass), the problem of confinement does not exist, since in all quark searches one naturally looks for an object with conventional particle attributes.

Although we do not want to assign continuous concepts such as position or momentum to the quark right from the beginning they might emerge in an appropriate large-structure limit, in accordance with the general spirit of the combinatorial approach. In the realm of strong interactions the only objects to which the concept of mass could be applied should be hadrons themselves. Since several quark model successes rest on the assumption that the quark is an orthodox particle with all its attributes, to substantiate the above considerations it is necessary to reproduce successful quark model relations using the concept of hadron mass alone. In the following section a number of such relations (and a few additional ones) are actually derived in this way. The basic ingredient of the approach is a combinatorial prescription for the spin dependence of hadron masses. This prescription is abstracted from the phenomenological approach of Żenczykowski (1986) and Törnqvist (1979) (based on the S-matrix ideas) as the leading term of hadron self-energy differences and in this paper is considered fundamental.

3. HADRON MASSES

The concept of quark mass will not be used below in the derivation of hadron mass relationships. The success of the quark model approach (Gell-Mann, 1962, 1964; Okubo, 1962) forces us to assume, however, that hadrons should be described as quantum states composed of quark–antiquark pairs (for mesons) and of three quarks (for baryons). The term "quark" describes here a spin-flavor index only. In accordance with the ideas of the preceding section, we do not assign momentum to a single quark (nor to a hadron constructed in this way: only through its correlation with the macroscopic world is the concept of continuous momentum thought to be eventually assignable to a hadron). No insight is proposed on the origin of flavor quantum number. "Ground-state" mesons transform as $1 \oplus 35$, "ground-state" baryons as 56 (i.e., symmetric) representation of $SU(6)_S$. To comply with the Pauli exclusion principle, the color quantum number should be assigned to quarks as well. It is, however, not necessarily related to QCD color: in this paper gluons are considered nonexistent. The only role of color is to provide an explanation for why one should work with the 56- (and not 20-) dimensional representation of $SU(6)_S$. From now on we shall therefore ignore it.

In earlier phenomenological studies (Żenczykowski, 1984, 1986; Törnqvist, 1979, 1982a,b; Törnqvist and Żenczykowski, 1984, 1986, 1987) the crucial ingredient was the consideration of the leading contribution to hadron self-energy coming from the symmetry-related set of two-hadron intermediate states. To estimate this contribution an assumption concerning the three-point hadron vertices was needed. This is the assumption of $SU(6)_W$-symmetry (Lipkin and Meshkov, 1965; Barnes *et al.*, 1965) (actually, the assumption is a little stronger, since the couplings 1-35-35, 35-35-35 or 1-56-56, 35-56-56 are assumed to be related by the quark model). Since for the vertices involving excited hadrons the use of a phenomenological model is at present necessary, we restrict our attention to "ground-state" mesons and baryons only.

The estimate of the self-energy contribution to the mass of hadron A due to its coupling to hadrons B and C (Fig. 1) involves the calculation of the square w_A^{BC} of the relevant $SU(6)_W$ Clebsch-Gordan coefficient, called henceforth a weight. As shown in Żenczykowski (1986) and Törnqvist and Żenczykowski (1984, 1986), the leading contribution from the loop shown in Fig. 1 is

$$m_A = C_0 + C_1 \sum_{B,C} w_A^{BC}(m_B + m_C) \tag{1}$$

where C_0, C_1 are unknown constants. C_0 may contain an additive dependence on flavor, but the spin dependence comes from the second term in equation (1) only. Formula (1) can be derived by expanding any specific expression for the loop of Fig. 1 to first order in the mass differences $m_B - \langle m_B \rangle$ and $m_C - \langle m_C \rangle$ (where $\langle m_B \rangle$ is some average mass of the multiplet to which B belongs) and neglecting all higher order terms. The only concepts used in (1) are discrete quantum-theoretic concepts (hadron masses, hadron and quark spins, as well as flavors).

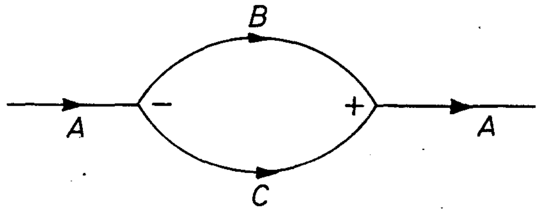

Fig. 1. The $A \to BC \to A$ loop. The minus (plus) denotes the clockwise (anticlockwise) ordering of particles in a vertex (CG coefficient $\langle A | BC \rangle$ or $\langle BC | A \rangle$).

At this point it is conjectured that equation (1) is more fundamental than its derivation and that it actually constitutes a proper combinatorial expression for hadron mass. Any additional terms in (1) cannot be obtained in any reliable way from the considerations like those of Żenczykowski (1984, 1986) or Törnqvist (1979, 1982a,b; Törnqvist and Żenczykowski, 1984, 1986, 1987). Rather, it is thought that such terms (if any) should be constructed within the combinatorial approach itself. Clearly, equation (1) defines the simplest class of such approaches.

Let the mesons A, B, and C belong to the 1- or 35-dimensional representation of $SU(6)_S$. The weight needed in equation (1) (we allow for the possibility of mixing among two states of $I=0$, $Y=0$; hence, in general, $A_1 \neq A_2$) is then

$$w^{BC}_{A_1A_2} = \sum_{\substack{m_A m_B \\ m_C}} \frac{1}{2S_A+1} \langle A_1 | BC \rangle \langle BC | A_2 \rangle \tag{2a}$$

with the $SU(6)_W$ Clebsch-Gordan coefficient given by

$$\langle A | BC \rangle \equiv \frac{(-1)^{W_A - 1}}{(2W_A+1)^{1/2}} \langle W_A m_A | W_B m_B W_C m_C \rangle (\bar{A}, B, C)_k \tag{2b}$$

$$(k = W_A + W_B + W_C)$$

The conventional S-spin values and their projections (S_A, m_A; S_B, m_B; S_C, m_C) uniquely determine the corresponding values of W-spin (W_A, W_B, W_C) needed in equation (2b). The flavor coupling is

$$\begin{aligned} (\bar{A}, B, C)_0 &= -2^{-1/2} F(\bar{A}, B, C) \\ (\bar{A}, B, C)_2 &= (3/2)^{1/2} F(\bar{A}, B, C) \\ (\bar{A}, B, C)_3 &= 3^{1/2} D(\bar{A}, B, C) \end{aligned} \tag{2c}$$

where

$$F(A, B, C) \equiv \mathrm{Tr}(ABC - CBA), \qquad D(A, B, C) \equiv \mathrm{Tr}(ABC + CBA)$$

$$(\bar{A})^k_n = A^n_k, \qquad (AB)^m_n = A^k_n B^m_k$$

and $A, B, \ldots$ are standard 3×3 meson matrices (Żenczykowski, 1986, Appendix C). The division into $SU(2)_W$ singlet and triplet depends of course on the choice of the auxiliary spin projection axis. Expression (2a) is, however, independent of this choice, as it should be if such an axis were to have no physical meaning at this stage.

Assume for the moment that m_B and m_C are all identical. The summation over B and C in equation (2a) can then be performed and if A is not a flavor-singlet vector particle, the result is independent of A. Thus (up to a possible additive flavor dependence of C_0), all particles but the vector flavor-singlet are degenerate. For the latter case the sum over B and C of w_A^{BC} weights is smaller, since the couplings of the $SU(6)_W$ singlet vanish by equation (2c). Then, the constant C_0 takes on a different value in the original derivation of Eq. (1) and, in general, the particle is not degenerate with other members of $1 \oplus 35$. We discuss possible remedies for this vector flavor-singlet problem a little further. Below it is accepted that the $I=0$, $Y=0$ vector sector (in which mixing with the flavor singlet occurs) cannot be described through equations (1) and (2) without some modifications. Thus, no formulas resulting from the application of equations (1) and (2) to this sector shall be discussed.

Let us now get rid of the unknown constant C_0 in equation (1) (together with its possible additive flavor dependence). By forming appropriate combinations of meson mass differences, the following five equations are obtained (two additional equations with parameters describing the sector mixed with the vector flavor singlet have been dropped):

$$(3\eta_8+\pi-4K)_{\text{ext}} = C_1[-2(3\eta_8+\pi-4K) + 2(3\omega_8+\rho-4K^*) - (\varphi-\omega)\cdot 4\sqrt{2}\sin 2\delta_V] \tag{3a}$$

$$(\rho-\pi)_{\text{ext}} = \frac{4}{3}C_1\left[2\rho-(\omega+\varphi)+(\varphi-\omega)\cos 2\delta_V + \frac{\eta+\eta'}{2}-\pi+\frac{\eta'-\eta}{2}\cos 2\delta_P\right] \tag{3b}$$

$$(K^*-K)_{\text{ext}} = C_1\frac{4}{3}\sqrt{2}(\sin 2\delta_P)\frac{\eta-\eta'}{2} \tag{3c}$$

$$\left(\frac{\eta-\eta'}{2}\sin 2\delta_P\right)_{\text{ext}} = C_1\cdot 2\sqrt{2}(K^*-K) \tag{3d}$$

and

$$\left(\frac{\eta+\eta'}{2}-\pi+\frac{\eta'-\eta}{2}\cos 2\delta_P\right)_{\text{ext}} = C_1(-4\pi+4\rho) \tag{3e}$$

In (3a)-(3e) the symbol of a particle stands for its mass, $\eta_8 \equiv c_P^2\eta + s_P^2\eta'$ and $\omega_8 \equiv c_V^2\varphi + s_V^2\omega$, where $c_P\,(c_V) \equiv \cos\theta_P\,(\cos\theta_V)$ and $s_P(s_V) \equiv \sin\theta_P\,(\sin\theta_V)$ and the transformation of states from the $SU(3)$-symmetric to the physical basis is effected by

$$\begin{bmatrix}|\omega\rangle\\|\varphi\rangle\end{bmatrix} = \begin{bmatrix}s_V & c_V\\ c_V & -s_V\end{bmatrix}\begin{bmatrix}|\omega_8\rangle\\|\omega_1\rangle\end{bmatrix} \tag{4a}$$

with $|\omega_8\rangle = 6^{-1/2}(u\bar{u} + d\bar{d} - 2s\bar{s})$ and $|\omega_1\rangle = 3^{-1/2}(u\bar{u} + d\bar{d} + s\bar{s})$ and similarly in the pseudoscalar sector with $|\omega\rangle, |\varphi\rangle \to |\eta'\rangle, |\eta\rangle$; $|\omega_8\rangle, |\omega_1\rangle \to |\eta_8\rangle, |\eta_1\rangle$; $c_V, s_V \to c_P, s_P$. For ideal mixing $\cos(\theta^{\text{ideal}}) = \sqrt{2/3}$ and $\sin(\theta^{\text{ideal}}) = 1/\sqrt{3}$, $|\varphi\rangle, |\eta\rangle \to -s\bar{s}$; $|\omega\rangle, |\eta'\rangle \to 1/\sqrt{2}(u\bar{u} + d\bar{d})$.

In (3a)-(3e) the mixing is described by δ, the angle of deviation from ideal mixing:

$$\theta_{P(V)} = \theta^{\text{ideal}}_{P(V)} + \delta_{P(V)} \tag{4b}$$

We assume in the vector sector for B and C mesons

$$\delta_V = 0 \qquad (3\omega_8 = 2\varphi + \omega) \tag{5a}$$

$$\rho = \omega \tag{5b}$$

$$\rho + \varphi - 2K^* = 0 \tag{5c}$$

Then, one obtains from (3a)

$$3\eta_8 + \pi - 4K = 0 \qquad (\text{if} \quad C_1 \neq -1/2) \tag{6a}$$

and after the diagonalization of the remaining equations (3b)-(3e) one gets

$$\begin{aligned}&\left[\rho - \pi + \frac{1}{\sqrt{3}}\left(\frac{\eta + \eta'}{2} - \pi + \frac{\eta' - \eta}{2}\cos 2\delta_P\right)\right]_{\text{ext}} \\ &\quad = \frac{4}{\sqrt{3}}C_1\left[\rho - \pi + \frac{1}{\sqrt{3}}\left(\frac{\eta + \eta'}{2} - \pi + \frac{\eta' - \eta}{2}\cos 2\delta_P\right)\right]\end{aligned} \tag{6b}$$

$$\begin{aligned}&\left[\rho - \pi - \frac{1}{\sqrt{3}}\left(\frac{\eta + \eta'}{2} - \pi + \frac{\eta' - \eta}{2}\cos 2\delta_P\right)\right]_{\text{ext}} \\ &\quad = -\frac{4}{\sqrt{3}}C_1\left[\rho - \pi - \frac{1}{\sqrt{3}}\left(\frac{\eta + \eta'}{2} - \pi + \frac{\eta' - \eta}{2}\cos 2\delta_P\right)\right]\end{aligned} \tag{6c}$$

$$\begin{aligned}&\left[K^* - K + \left(\frac{2}{3}\right)^{1/2}\frac{\eta - \eta'}{2}\sin 2\delta_P\right]_{\text{ext}} \\ &\quad = \frac{4}{\sqrt{3}}C_1\left[K^* - K + \left(\frac{2}{3}\right)^{1/2}\frac{\eta - \eta'}{2}\sin 2\delta_P\right]\end{aligned} \tag{6d}$$

$$\begin{aligned}&\left[K^* - K - \left(\frac{2}{3}\right)^{1/2}\frac{\eta - \eta'}{2}\sin 2\delta_P\right]_{\text{ext}} \\ &\quad = -\frac{4}{\sqrt{3}}C_1\left[K^* - K - \left(\frac{2}{3}\right)^{1/2}\frac{\eta - \eta'}{2}\sin 2\delta_P\right]\end{aligned} \tag{6e}$$

Since C_1 should be positive [it corresponds (Törnqvist and Żenczykowski, 1984, 1986), e.g., to the first derivative of the negative and rising shift function], it follows from (6a), (6c), and (6e) that

$$\pi + 3\eta_8 - 4K = 0 \tag{7a}$$

$$\rho - \pi = \frac{1}{\sqrt{3}}\left(\frac{\eta + \eta'}{2} - \pi + \frac{\eta' - \eta}{2}\cos 2\delta_P\right) \tag{7b}$$

$$K^* - K = \left(\frac{2}{3}\right)^{1/2} \sin 2\delta_P \frac{\eta - \eta'}{2} \tag{7c}$$

From (6b) and (6d) we have $C_1 = 3^{1/2}/4$ (provided $\rho - \pi$ or $K^* - K \neq 0$). If $\eta' > \eta$ and $K^* > K$, it follows from (7c) that $\delta_P < 0$, in agreement with the experimental sign. For $\delta_P = -45°$ ("perfect" mixing), we get for the left- and right-hand sides of (7b) and (7c) the numbers gathered in Table I. The case with particle symbols standing for mass squares is given there as well. The mixing of the light pseudoscalar states (composed of u, d, s quarks) with the heavy ones (made of $c, \ldots$) is expected to increase the rhs in (7b) and (7c), thus improving the agreement with experiment. Conversely, if δ_P is forced to be equal to zero, the particle of hidden strangeness decouples from ρ and π, which should diminish the rhs of (7b). It is interesting to look at a simplified situation in which internal pseudoscalar mesons composed of new quark types (q) are assumed to be unmixed with η, η'. The weights of the contributions from mesons containing quark q to the light meson masses are given in Table II.

From Table II it follows that heavy flavors do not contribute to the mass differences of light mesons: the q-quark contributions to the left- and right-hand sides of (7a)–(7c) are identically zero. A detailed study of the mixing problem for $N_f > 3$ is beyond the scope of this paper, due to the rapidly increasing (with N_f) number of mixing angles and the unknown character of flavor symmetry breaking in vertices.

It should be noted that the vector flavor-singlet problem depends on the number of flavors N_f: the $SU(N_f)$ singlet state contains smaller and smaller admixture of the $SU(2)$ [$SU(3)$] singlet when $N_f \to \infty$. This suggests a possible way of dealing with the singlet problem for any finite number of flavors. For the state $(1/\sqrt{2})(u\bar{u} + d\bar{d})$ to be an approximate eigenstate of the infinite mass matrix, it is sufficient that the ratio of sums over

Table I. Comparison of Eqs. (7b) and (7c) with Experiment

	Mass		Mass squared	
	lhs	rhs	lhs	rhs
Eq. (7b)	0.627	0.355	0.566	0.341
Eq. (7c)	0.396	0.167	0.550	0.252

Table II. Weights of Contributions from Internal Mesons Containing "Heavy" Quark q[a]

A \ B, C					
	$(u\bar{q})(\bar{u}q)^*$	$(u\bar{q})^*(\bar{u}q)^*$	Total		
π	2	2	4		
	$(u\bar{q})(\bar{u}q)$	$(u\bar{q})(\bar{u}q)^*$	$(u\bar{q})^*(\bar{u}q)^*$	Total	
ρ	1/3	4/3	7/3	4	
	$(u\bar{q})(q\bar{s})^*$	$(u\bar{q})^*(q\bar{s})$	$(u\bar{q})^*(q\bar{s})^*$	Total	
K	1	1	2	4	
	$(u\bar{q})(q\bar{s})$	$(u\bar{q})(q\bar{s})^*$	$(u\bar{q})^*(q\bar{s})$	$(u\bar{q})^*(q\bar{s})^*$	Total
K^*	1/3	2/3	2/3	7/3	4
	$(u\bar{q})(\bar{u}q)^*$	$(s\bar{q})(\bar{s}q)^*$	$(u\bar{q})^*(\bar{u}q)^*$	$(s\bar{q})^*(\bar{s}q)^*$	Total
$\begin{bmatrix} \eta_{uu} & \eta_{us} \\ \eta_{su} & \eta_{ss} \end{bmatrix}$	$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$	$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$	$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

[a] Pseudoscalar (vector) mesons are denoted by $(u\bar{q})$ $[(\bar{u}q)^*]$ if the "light" quark is nonstrange and by $(s\bar{q})$ $[(\bar{s}q)^*]$ if it is strange;

$$\eta_{uu} = (\eta + \eta')/2 + \cos 2\delta_P\,(\eta' - \eta)/2,$$

$$\eta_{su} = \eta_{us} = \sin 2\delta_P\,(\eta' - \eta)/2, \text{ and } \eta_{uu} + \eta_{ss} = \eta + \eta'.$$

$k > 2(u, d)$ of the off-diagonal and of the diagonal contributions from the internal $(u\bar{q}_k) + (\bar{u}q_k)$ pairs be close to zero. This can be achieved through an appropriate breaking of vertex symmetry. Another possibility of dealing with the vector flavor-singlet problem was mentioned in Törnqvist (1985): it requires the consideration of the "excited" states (conjugated by C-parity to the "ground" states), in addition to the "ground" states themselves. At present only a phenomenological treatment of the involved vertices is possible, however.

Since the vector flavor-singlet problem is absent from the baryon sector, it is interesting to examine the "heavy" quark contribution in this sector. For the "light" quarks the $SU(6)_W$ Clebsch-Gordan coefficient for baryon B going into meson M and baryon A (meson first convention) is given by

$$\langle B \mid MA \rangle = \frac{(-1)^{W_B - 3/2}}{(2W_B + 1)^{1/2}} \langle W_B m_B \mid W_M m_M W_A m_A \rangle [\bar{B}, M, A]_{kl}$$

$$(k \equiv W_B + W_A, \quad l \equiv W_M) \tag{8a}$$

with the flavor couplings:

$$[\bar{B}, M, A]_{11} = \frac{1}{\sqrt{3}}[\mathrm{Tr}(M)\,\mathrm{Tr}(\bar{B}A) - \mathrm{Tr}(\bar{B}AM) - 5\,\mathrm{Tr}(\bar{B}MA)]$$

$$[\bar{B}, M, A]_{10} = -\mathrm{Tr}(M)\,\mathrm{Tr}(\bar{B}A) + \mathrm{Tr}(\bar{B}AM) - \mathrm{Tr}(\bar{B}MA)$$

$$[\bar{B}_8, M, A_{10}]_{21} = -\frac{2\sqrt{2}}{(s_{bij})^{1/2}} A_{bij} \bar{B}^j_l M^b_a \varepsilon^{ila}$$

$$[\bar{B}_{10}, M, A_8]_{21} = \frac{2\sqrt{2}}{(s_{bij})^{1/2}} \bar{B}^{bij} M^a_b A^l_j \varepsilon_{ial} \tag{8b}$$

$$[\bar{B}, M, A]_{31} = \frac{(30)^{1/2}}{(s_{aij}s_{bij})^{1/2}} \bar{B}^{aij} A_{bij} M^b_a$$

$$[\bar{B}, M, A]_{30} = \frac{3\sqrt{2}}{(s_{aij}s_{bij})^{1/2}} \bar{B}^{aij} A_{bij} M^b_a$$

where

$$s_{ijk} = \begin{cases} 1 & \text{for} \quad i = j = k \\ 6 & \text{for} \quad i \neq j \neq k \neq i \\ 3 & \text{in remaining cases} \end{cases}$$

with the standard assignment for meson (M) and baryon (B, A) matrices (Żenczykowski, 1986, Appendix C). The weights needed in equation (1) are

$$w_B^{MA} = \sum_{\substack{m_A m_B \\ m_M}} \frac{1}{2S_{\mathcal{B}} + 1} \langle B \mid MA \rangle \langle MA \mid B \rangle \tag{8c}$$

In Żenczykowski (1986) it is shown that the contribution from the "ground"-state intermediate hadrons composed of u, d, s quarks preserves the Gell-Mann–Okubo formula for octet and the equal spacing rule for the decouplet. The signs of $\Delta - N$ and $\rho - \pi$ splittings were shown to be related and identical. Equations (1) and (8c) in addition to $SU(3)$ mass formulas lead to

$$\Sigma^* - \Sigma - (\Xi^* - \Xi) = 0 \tag{9a}$$

$$\Sigma - 3\Lambda - 2\Delta + 2N + 2\Sigma^* = 0 \tag{9b}$$

with (9a) being the $SU(6)$ relation of Gürsey and Radicati (1964) and Pais (1964) and (9b) the formula of de Rújula *et al.* (1975) for the $(\Sigma - \Lambda)/(\Delta - N)$ ratio of mass differences.

It may be checked after lengthy but straightforward calculations [using a suitably modified equation (8b) or Table 1 of Żenczykowski (1986)] that a complete set of ground-state intermediate hadrons containing an additional quark q does not contribute to the mass combinations (9a), (9b) of external baryons. This cancellation occurs independently of the nature of the mass relations between the q-containing hadrons themselves. On the other hand, the Gell-Mann–Okubo and the equal spacing $SU(3)$ formulas are fulfilled for external lines provided the following equalities hold for the internal lines as well:

$$\begin{aligned} \{uu\}q+\{ss\}q-2\{us\}q&=0 \\ \{uuq\}^*+\{ssq\}^*-2\{usq\}^*&=0 \end{aligned} \tag{9c}$$

where $\{uu\}q(\{uuq\}^*)$ denotes the mass of the spin-1/2 state with up and/or down quarks symmetrized (the mass of the fully symmetric in flavor spin-3/2 state). If (9a)-(9c) and the $SU(3)$ mass formulas are fulfilled for the internal lines, then (9c) for the external baryons is also fulfilled. Apart from a degenerate case [a specific single value of C_1 (baryon sector)] (9c) results also by solving (1) for $N_f=4$ [and the $SU(8)_W$ vertex symmetry].

Equation (1) can also be applied to the question of isospin-violating mass differences of hadrons. Such calculations have recently been done for the ground-state baryons (Törnqvist and Żenczykowski, 1987), where there are ten linearly independent mass differences. It has been shown that equation (1) leads to: (1) six sum rules for baryon masses, which follow from the assumed $SU(6)$ symmetry and are satisfied by other models as well; (2) additional prescriptions for four combinations of baryon masses, which, together with a phenomenological estimate of contributions from other possible sources, describe the observed pattern of isospin-violating mass differences very well.

A shortened version of Table 1 of Törnqvist and Żenczykowski (1987) obtained after neglecting possible dependence of C_0 in (1) on the third component of isospin is given here as Table III. Table III reveals that the contributions of equation (1) alone suffice to predict, in agreement with

Table III. Comparison of Experiment with Predicted Ratios of the Combinations of Isospin-Violating Mass Differences That Are Sensitive to the Dynamical Input

Ratio of mass combinations	Prediction[a]	Experiment[b]
$\dfrac{p-n-1/6[\Sigma^+-\Sigma^-+2(\Sigma^{*+}-\Sigma^{*-})]}{p-n+1/6[\Sigma^+-\Sigma^-+2(\Sigma^{*+}-\Sigma^{*-})]}$	-0.25 ± 0.04	-0.19 ± 0.06
$\dfrac{\Sigma^+-\Sigma^--(\Sigma^{*+}-\Sigma^{*-})}{p-n+1/6[\Sigma^+-\Sigma^-+2(\Sigma^{*+}-\Sigma^{*-})]}$	$+0.23\pm0.09$	$+0.54\pm0.18$

[a] Equation (1).
[b] Electromagnetic contribution subtracted.

experiment (after subtracting the electromagnetic contribution), two out of three ratios of mass combinations of point 2 above. One still lacks part of the necessary experimental input required by equation (1) to compute the numerator of the third ratio. The agreement seen in Table III is very interesting: three mass differences that customarily are thought to be of different dynamical origins are apparently correctly related by a single group-theoretic prescription. [One of the mass combinations considered, namely $p-n+1/6[\Sigma^{+}-\Sigma^{-}+2(\Sigma^{*+}-\Sigma^{*-})]$ measures what is usually thought to arise from the quark mass difference m_u-m_d.] It should be recalled that in the phenomenological study of Törnqvist and Żenczykowski (1987) leading to Table III all the dependence on the momentum variable has been totally ignored. Yet the resulting numbers are in surprisingly good agreement with experiment.[3] The success achieved under the assumption of neglecting the momentum altogether may be interpreted as corroborating the "discrete" spirit of the combinatorial approach, and the need to introduce a discrete "predecessor" of momentum in such a way that would not violate the successful mass predictions of this section.

4. VERTICES

Data on baryon magnetic moments [supplemented with the assumption of vector meson dominance (Schwinger, 1967; Żenczykowski, 1985)] and other data on hadronic couplings (Arndt *et al.*, 1979) indicate that $SU(6)_W$ and flavor $SU(3)$ symmetries are broken in a peculiar nonadditive way (Lipkin, 1983; Dicus and Teplitz, 1985). This experimentally observed pattern of vertex symmetry breaking has not yet been explained in any scheme. Thus it is of interest to examine the predictions of the group-theoretic approach modeled upon the treatment of masses of the previous section. Below the results of such a calculation are briefly sketched.

Consider the diagrams of Fig. 2, which directly describe the group-theoretic structure of the baryon-meson-baryon (BMA) vertex and are analogous to equation (2a) or (8c). Each vertex component in Fig. 2a corresponds to an $SU(6)_W$ Clebsch-Gordan coefficient of (2b) or (8a). The direction of the arrows (incident $\leftrightarrow$ outgoing) and vertex orientation correspond to the bra-ket description and meson first convention of the $SU(6)_W$ Clebsch-Gordan coefficients, respectively. Relative normalization and phases in (2b) and (8) have been chosen consistently so that a simple product of CG coefficients possesses the required $SU(6)_W$ symmetry properties after the summation over internal states is carried out. The contributions

[3]The predicted ratio of $\{p-n+1/6[\Sigma^{+}-\Sigma^{-}+2(\Sigma^{*+}-\Sigma^{*-})]\}/(\Delta-N)$ is 30% larger than the experimental result. This might indicate the necessity of considering flavor symmetry breaking in the vertices.

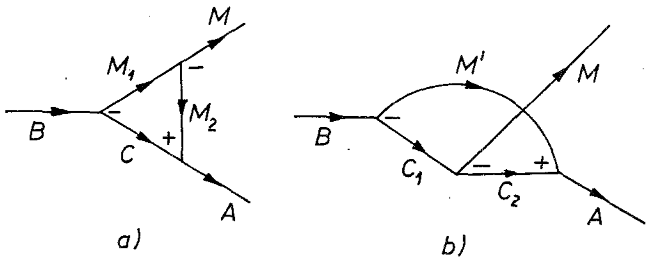

Fig. 2. The group-theoretic structure of the $B \to MA$ vertex: (a) meson M emitted from internal meson line, (b) meson M emitted from internal baryon line. The minus (plus) near vertices denotes the clockwise (anticlockwise) ordering of particles in a vertex.

from Fig. 2a and 2b are therefore, respectively,

$$L^{(a)}_{B\to MA}(M_1, M_2, C) = \langle B|M_1C\rangle\langle M_1|MM_2\rangle\langle M_2C|A\rangle \qquad (10a)$$

$$L^{(b)}_{B\to MA}(M', C_1, C_2) = \langle B|M'C_1\rangle\langle C_1|MC_2\rangle\langle M'C_2|A\rangle \qquad (10b)$$

To calculate (10a) and (10b), one needs, besides equations (2b) and (8a),

$$\langle MA|B\rangle = \frac{(-1)^{W_B-3/2}}{(2W_B+1)^{1/2}}\langle W_Mm_MW_Am_A|W_Bm_B\rangle[\bar{A}, \bar{M}, B]_{kl}$$

$$(k \equiv W_B + W_A, \quad l \equiv W_M) \qquad (11)$$

For any $M \in 35$ of $SU(6)_W$ (but not for the singlet) one has

$$\sum_{\substack{M_{1,2}\in 1\oplus 35\\ C\in 56}} L^{(a)} = 6\langle B|MA\rangle, \qquad \sum_{\substack{M'\in 1\oplus 35\\ C_{1,2}\in 56}} L^{(b)} = 18\langle B|MA\rangle$$

Thus, if the masses corresponding to internal lines are degenerate, so that such summations can actually be performed, the resulting BMA vertex is $SU(6)_W$-symmetric.

Introducing the breaking of the $SU(6)_W$-vertex symmetry through mass differences of internal lines only and estimating it (as before in the mass sector itself) to first order in mass differences, one gets expressions of the following structure [$M \in 35$ of $SU(6)_W$]:

loop contribution

$$\begin{aligned}&= L_0\langle B|MA\rangle\\ &\quad + L_1\langle B|MA\rangle(w^{M_1}_{BMA}m_{M_1} + w^{M_2}_{BMA}m_{M_2} + w^{C}_{BMA}m_C\\ &\quad + w^{M'}_{BMA}m_{M'} + w^{C_1}_{BMA}m_{C_1} + w^{C_2}_{BMA}m_{C_2})\\ &\quad + \text{possible Zweig-rule-violating term}\end{aligned} \qquad (12)$$

where L_0 and L_1 are constants and $w_{BMA}^{M_1}$, etc., are weights calculated on a computer. It turns out that Zweig-rule-violating terms vanish if $K^* - K = 4/3(\Sigma^* - \Sigma)$, what is not far from experiment. Such terms were therefore neglected. From equation (12), upon assuming the dominance of vector mesons (Schwinger, 1967; Żenczykowski, 1985), one can derive a set of formulas for baryon magnetic moments and magnetic transitions. There are two free parameters in these formulas: the size μ (adjusted to fit μ_P) of the $SU(6)_W$-symmetric term (from freedom in L_0), which may be argued to be approximately determined by $2m_N/m_\rho$ (Schwinger, 1967; Żenczykowski, 1985) and the size C of the correction (from L_1). Using for simplicity perfect mixing for pseudoscalar and ideal mixing for vector mesons and inserting physical hadron masses into equation (12), one gets finally (the following equations with all the weights and thus with all the mass dependences shown explicitly are complicated and their full presentation here is redundant)

$$
\begin{aligned}
\mu_p = \mu, \qquad & \mu_n = -2/3\mu \\
\mu_{\Sigma^+}/\mu &= 8/9(1+8C) + 1/9\varepsilon(1+9.3C) \\
\mu_{\Sigma^-}/\mu &= -4/9(1+8.2C) + 1/9\varepsilon(1+9.3C) \\
\mu_\Lambda/\mu &= -1/3\varepsilon(1+9.4C) + 0.1C \\
\mu_{\Xi^0}/\mu &= -2/9(1+12.1C) - 4/9\varepsilon(1+16.5C) \\
\mu_{\Xi^-}/\mu &= 1/9(1+11.8C) - 4/9\varepsilon(1+16.5C) \\
\mu_{\Delta\to p\gamma}/\mu &= 2/3\sqrt{2}(1+1.38C), \qquad \varepsilon \equiv m_\rho/m_\varphi
\end{aligned}
\tag{13}
$$

It turns out that no choice of parameter C in equations (13) can explain the observed pattern of nonadditivities in baryon magnetic moments and magnetic transitions (Lipkin, 1983; Schwinger, 1967; Żenczykowski, 1985). Thus, the combinatorial scheme as considered so far is able to describe properly the pattern of hadron masses, but fails when one attempts to apply it in the most naive way to the description of vertex symmetry breaking. Success in the description of masses and failure in the description of vertices is a feature of all other "successful" contemporary schemes as well.

It should be noted that the attempt of this section is based on the simplest possible guess concerning the algebraic structure of vertices, justified by the success of equation (1) only. Within the S-matrix approach the W-spin projection axis cannot be identical for all three subvertices of the diagrams corresponding to Fig. 2, as opposed to the situation encountered in Fig. 1. Thus, the introduction of momentum of intermediate particles into the description of vertex symmetry breaking may seem necessary. Clearly, if the general idea of this paper is a sound one, momentum should be introduced in a way compatible with the discrete spirit of the approach. The twistor approach does not lead, however, to the conventional classification of leptons and quarks (Penrose, 1977; Perjés, 1977). Thus, the question of how momentum should be introduced is still an open problem.

5. CONCLUDING REMARKS

In standard approaches quarks are thought of as pointlike particles confined into extended hadrons. Apart from the interaction of quarks within hadrons (which, among other effects, leads to quark confinement), there are also interactions of hadrons themselves, leading to hadronic self-energy shifts and other hadron-level effects. All this physics takes place in ordinary space and equation (1) constitutes the leading term of the extremely complicated contribution from the "hadronic cloud." This was the point of view adopted in Żenczykowski (1986).

In this paper group-theoretic discrete concepts are considered fundamental and it is thought that concepts such as continuous space and momentum and, consequently, the field-theoretic description of quark degrees of freedom in hadrons should emerge in an appropriate large-structure limit only. Equation (1) is then thought to provide a connection between the S-matrix approach to hadron masses through self-energies (with classical momentum constituting one of the primary concepts of the approach) and the purely algebraic approach to hadron masses from which the naive quark model has originated. The question of a possible correspondence between other contributions of the S-matrix approach and the algebraic approach remains of course open.

Whether the interpretation advocated in this paper should be considered as more acceptable than the orthodox point of view adopted in Żenczykowski (1986) can be decided only by further attempts to develop algorithms based solely on discrete concepts and capable of linking together various experimental facts.

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